The initial mass function of stellar clusters

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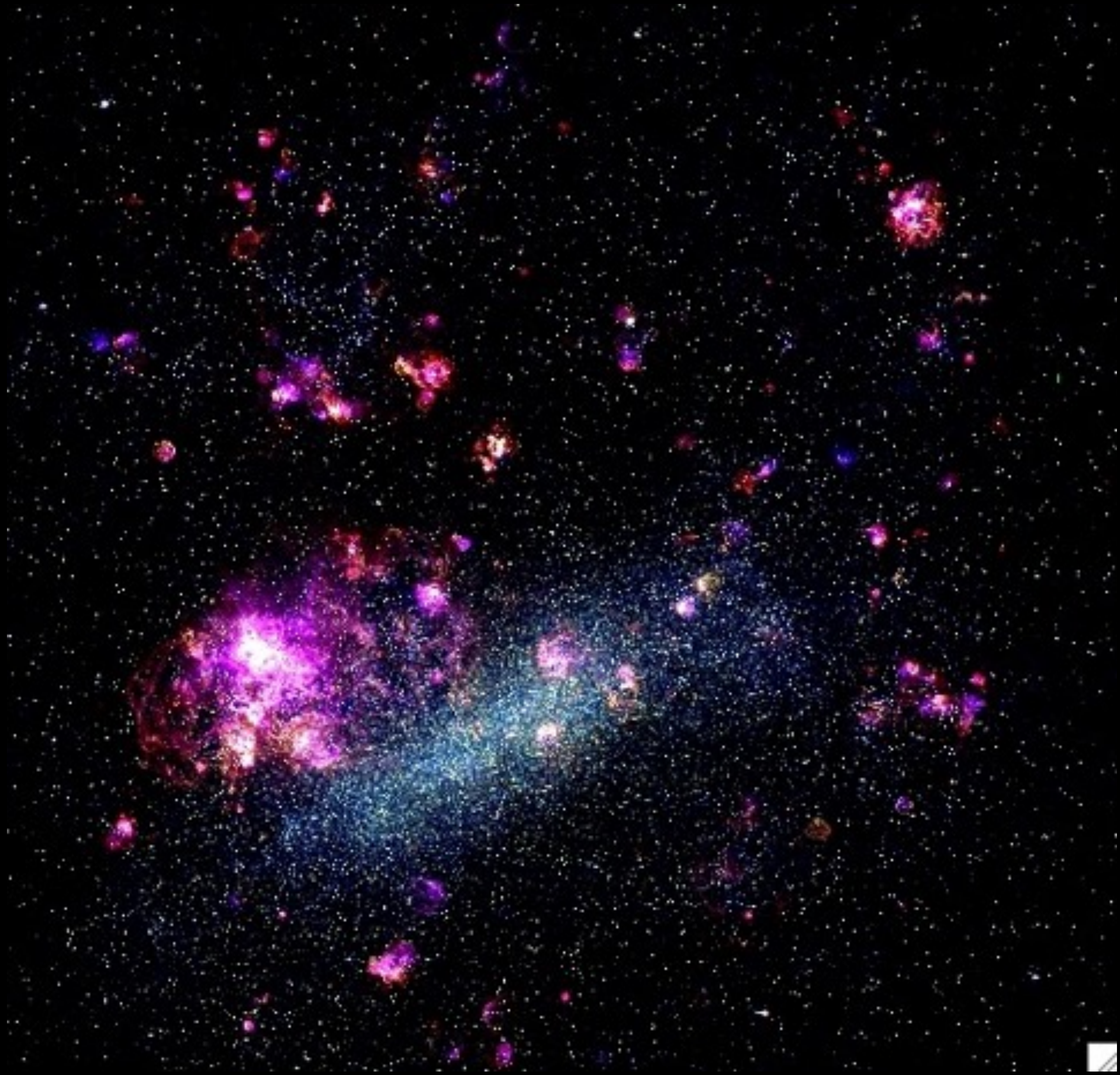


Supervised by Prof. Shantanu Basu
Collaboration with Sophia Lianou

What is the Initial Mass Function (IMF)?

The distribution of stellar masses at birth into different mass intervals is called the initial mass function.

So what do I do with the IMF?

Modified Lognormal Power-law (MLP) function

$$f(m) = \frac{\alpha}{2} \exp\left(\alpha\mu_0 + \alpha^2\sigma_0^2/2\right) m^{-(1+\alpha)} \times \text{erfc}\left(\frac{1}{\sqrt{2}}\left(\alpha\sigma_0 - \frac{\ln m - \mu_0}{\sigma_0}\right)\right), \quad m \in [0, \infty),$$

Basu and Jones 2004

- Most commonly used IMF models are **piecewise functions**.
- Study the **MLP** distribution function as a **single function** to describe the underlying mass distribution i.e. IMF.


m
0.2 M⊙
1 M⊙
10 M⊙
IMF MODEL

So..How do you measure stellar masses?

Direct Measurement!

Eclipsing Binaries + Kepler's laws

www.eso.org

Some Terminology

Luminosity =  **Power** = **Energy/ time**

60 W 60 W 60 W

 d1  d2  d3

1 2 3

Apparent magnitude = 

↑↓

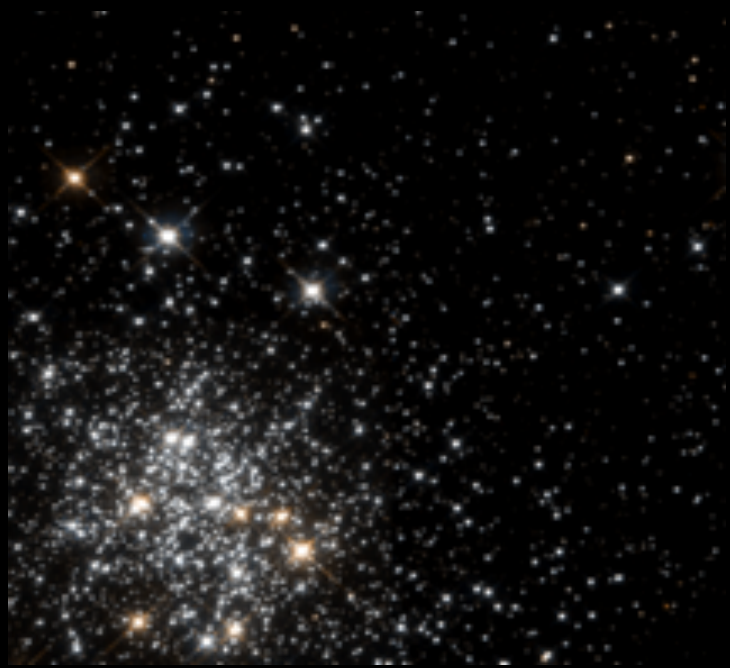
Absolute Magnitude = 

60 W  3

80 W  2 Hot

100 W  1

Mass-Luminosity Relation(MLR)



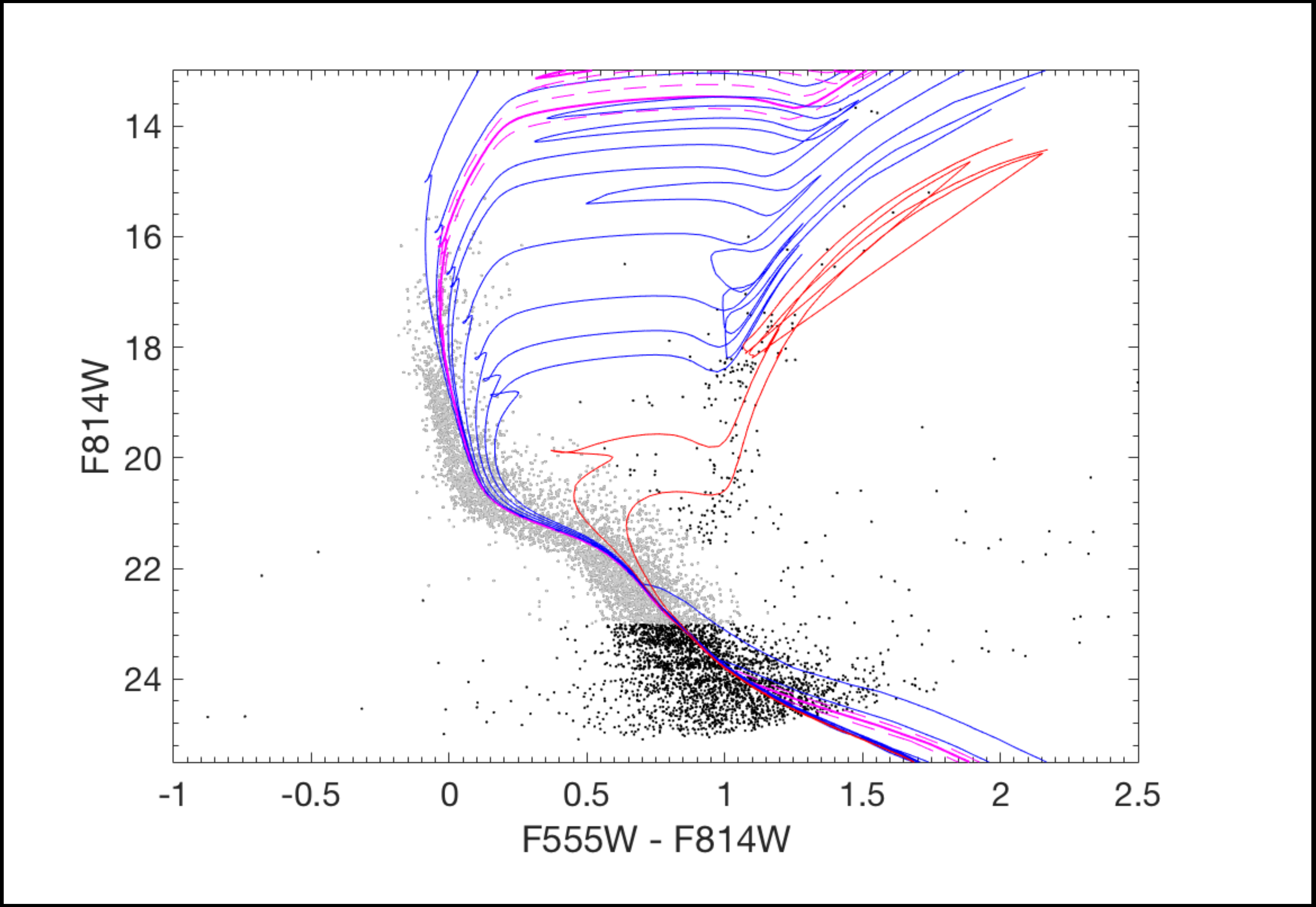
Resolved population

Luminosity/ Absolute magnitude

Isochrone Models

Mass

Color-Magnitude Diagram and Isochrones



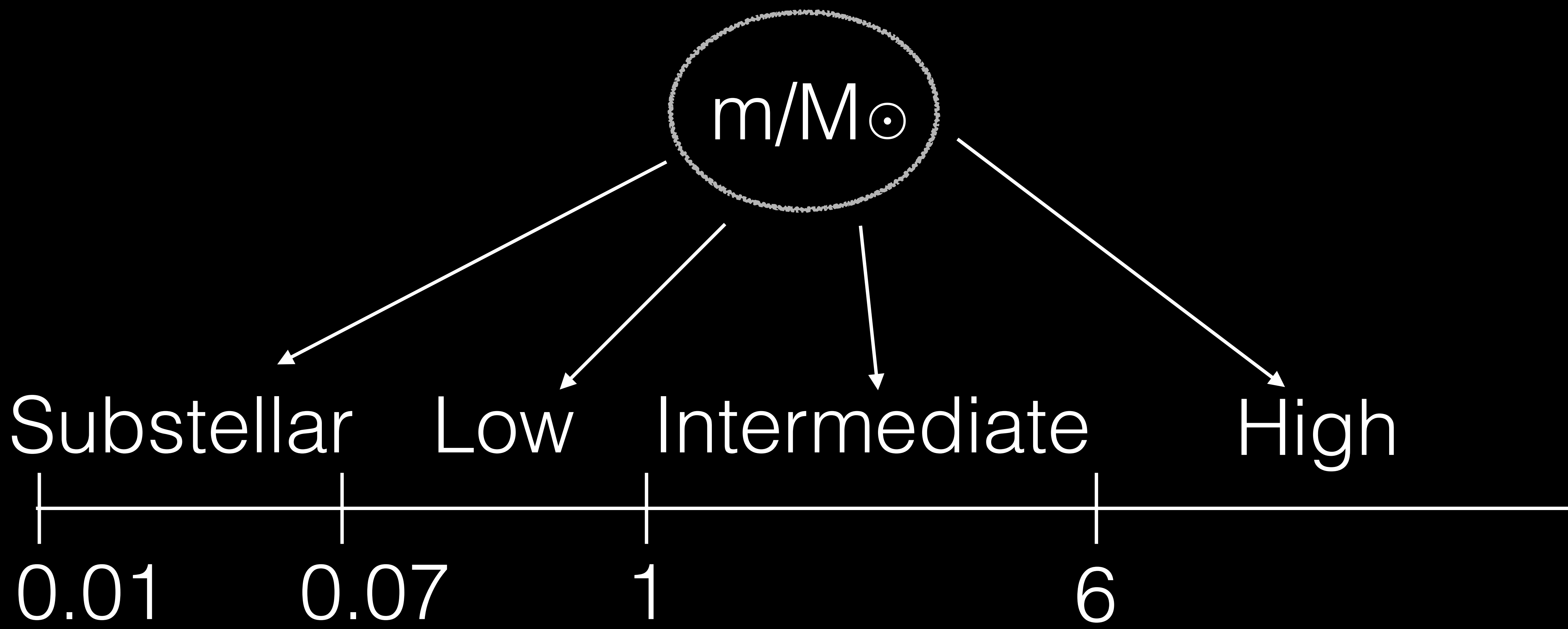
Apparent Magnitude

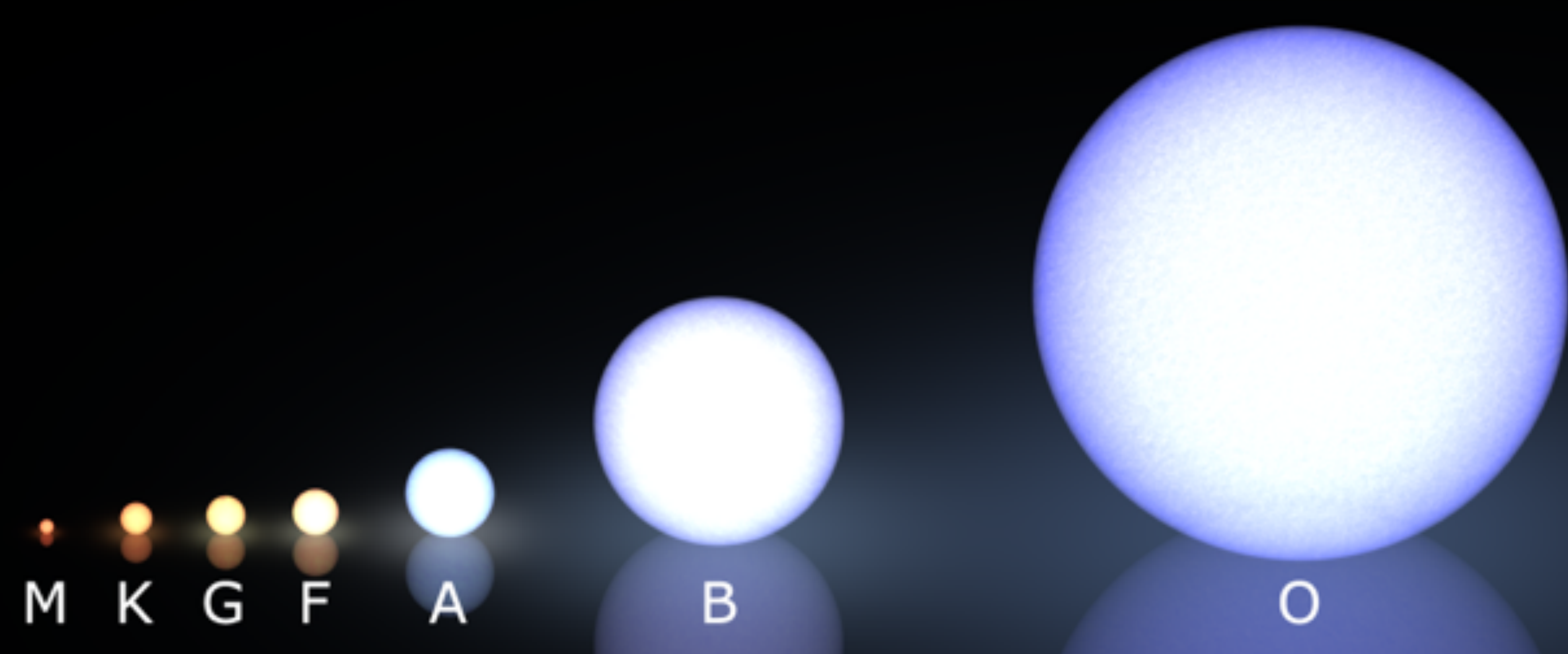
Color

Age of the cluster: 51 million years

A look at the stellar mass regime!

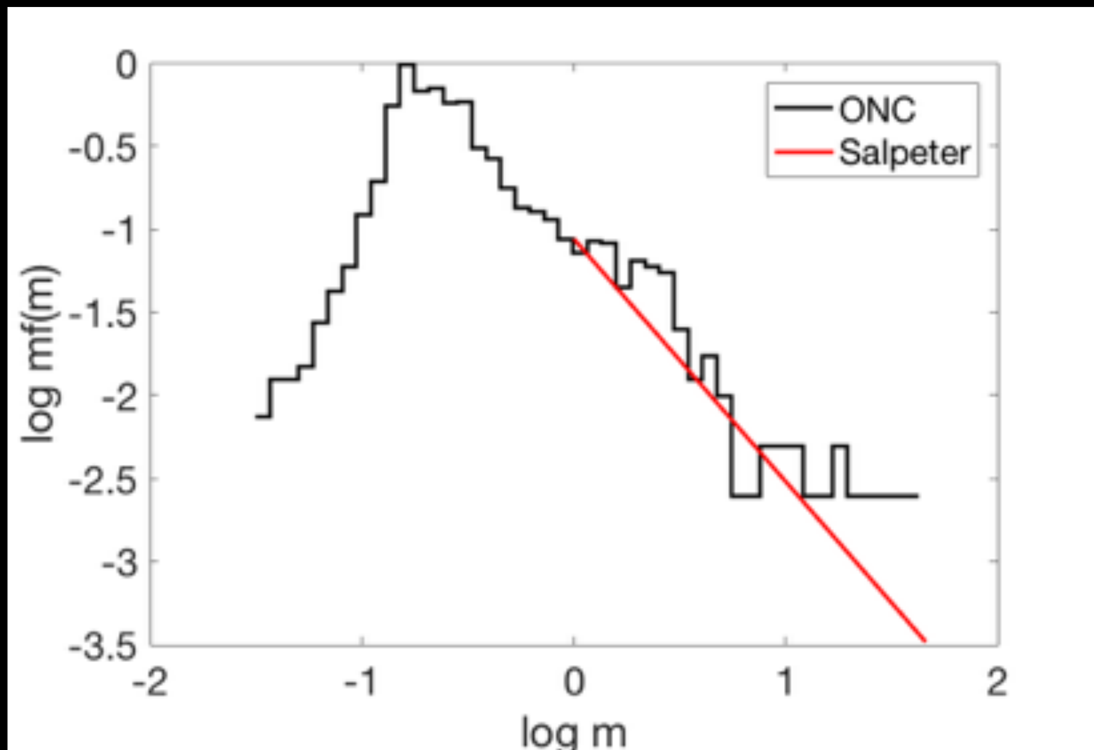
Stellar Mass Regime



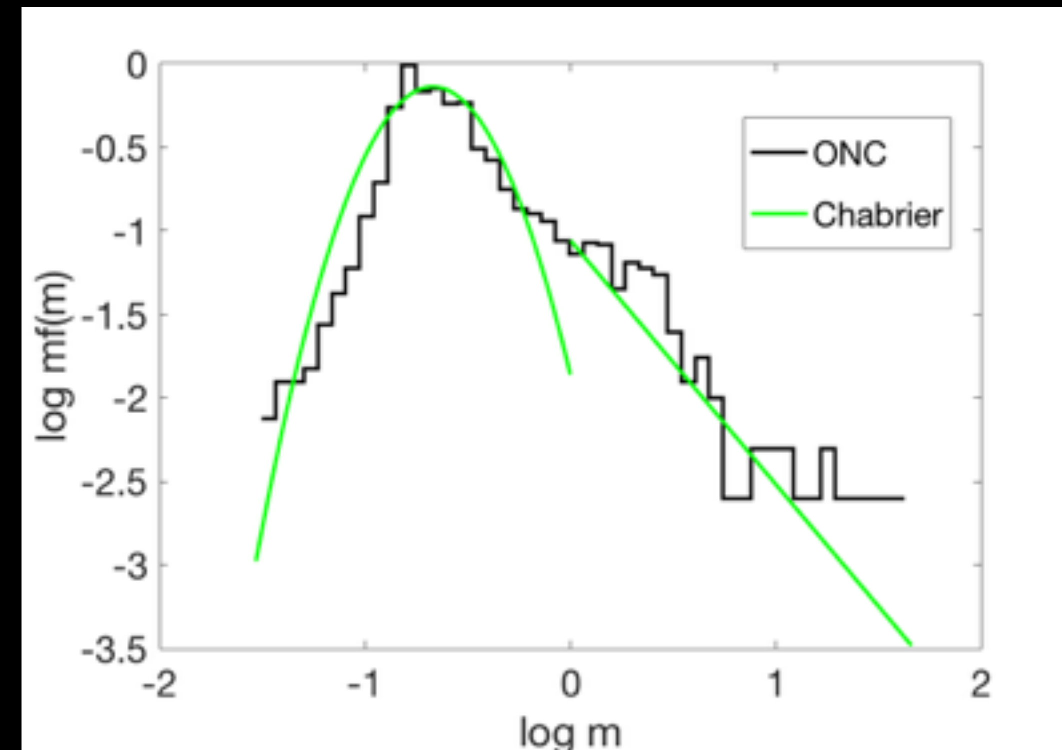


What IMF models are available in the market?

IMF Models



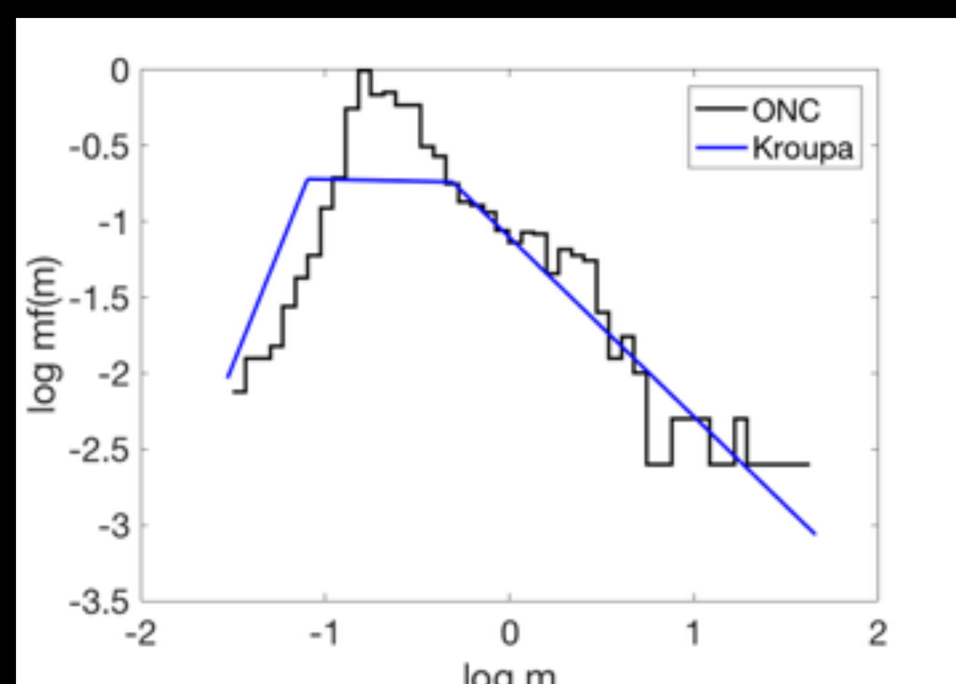
Salpeter IMF: power-law



Chabrier IMF: lognormal + power-law

$$dN/d\ln m \propto m^{-1.35}$$

$> 1\ M_\odot$



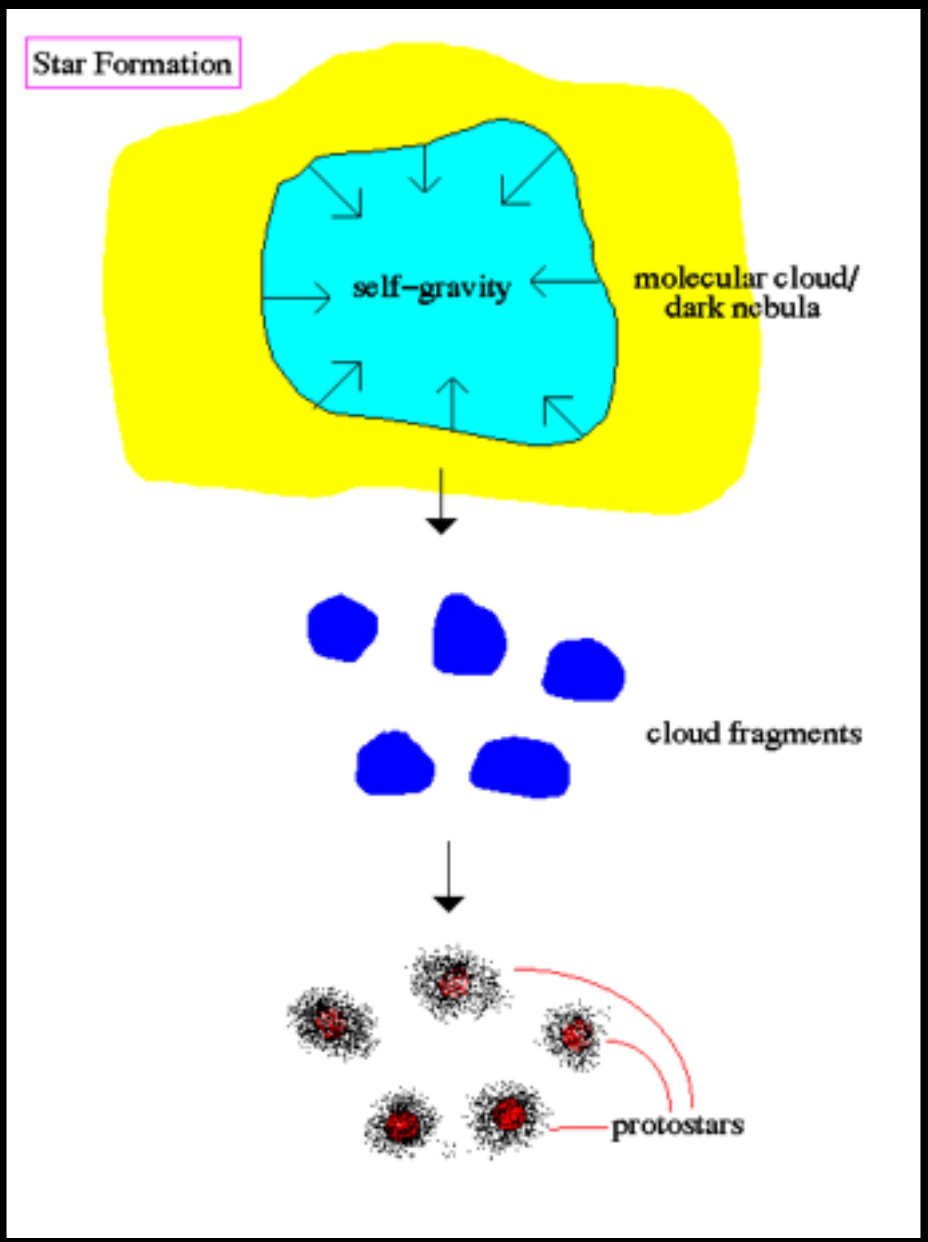
Kroupa IMF: three broken power-laws

I know.. Astronomers like things a little complicated

Modified Lognormal Power-law Probability Distribution Function

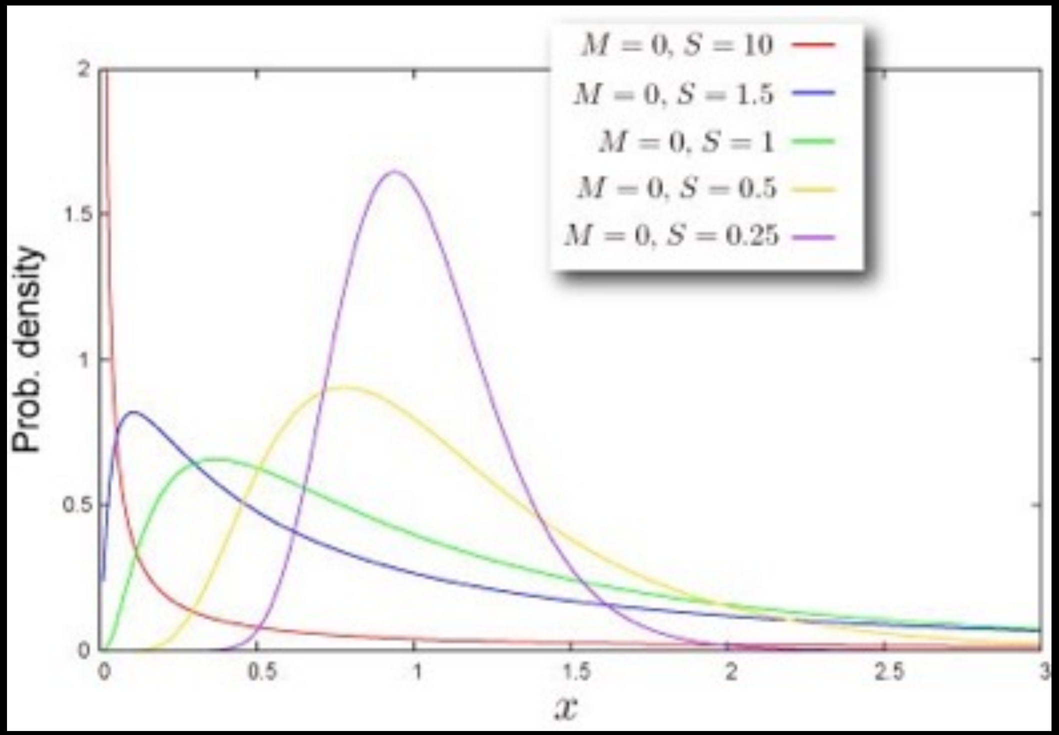
Initial lognormal distribution

Processes that determine the mass of a star: Gravity, Turbulence, Magnetic Fields, outflows…



$$m = f_1 \times f_2 \times \ldots\ldots \times f_N.$$

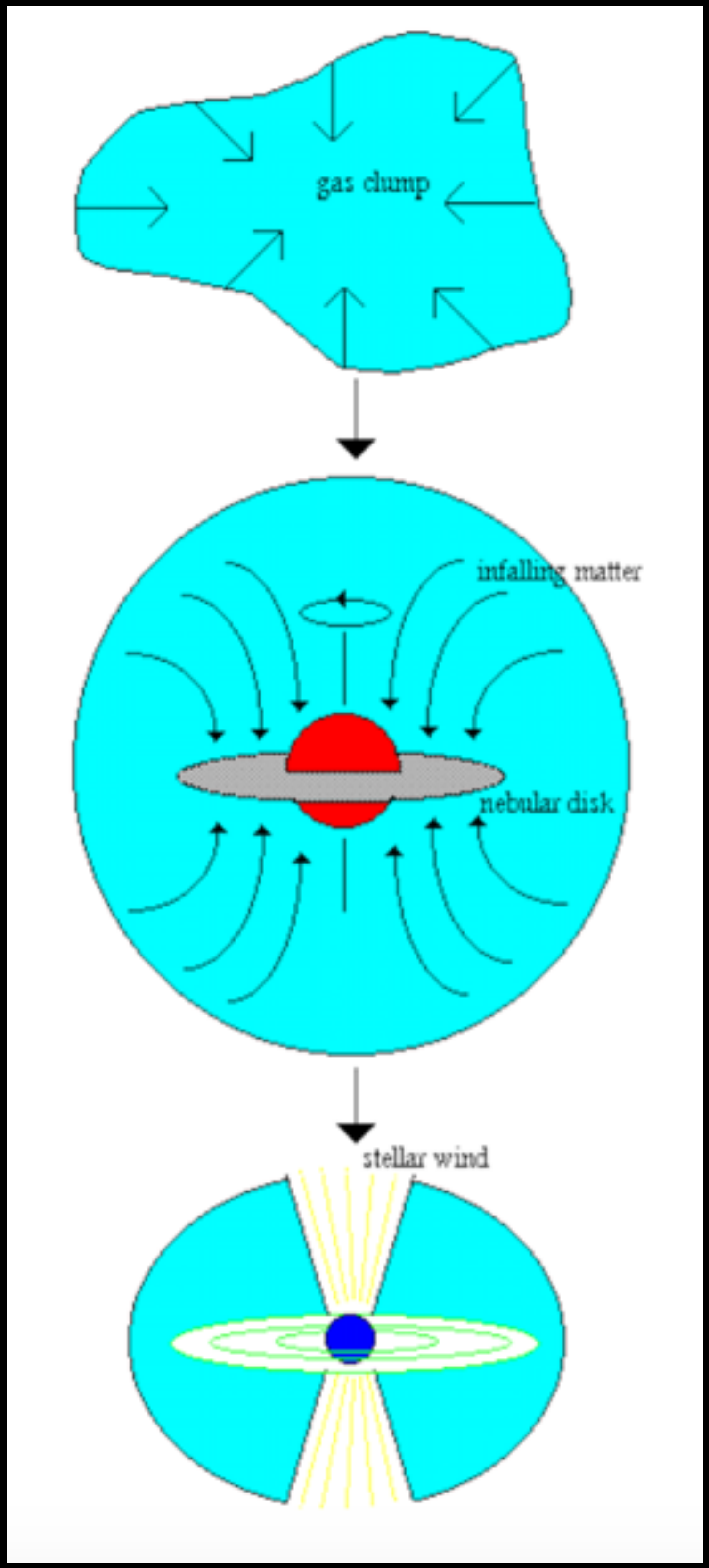
Central Limit Theorem



Source: http://abyss.uoregon.edu/~js/ast122/lectures/lec13.html

Basu et al. 2015

Accretion!



$$\frac{dm}{dt} = \gamma m\,,\, m(t) = m_0 \exp(\gamma t)\,,$$

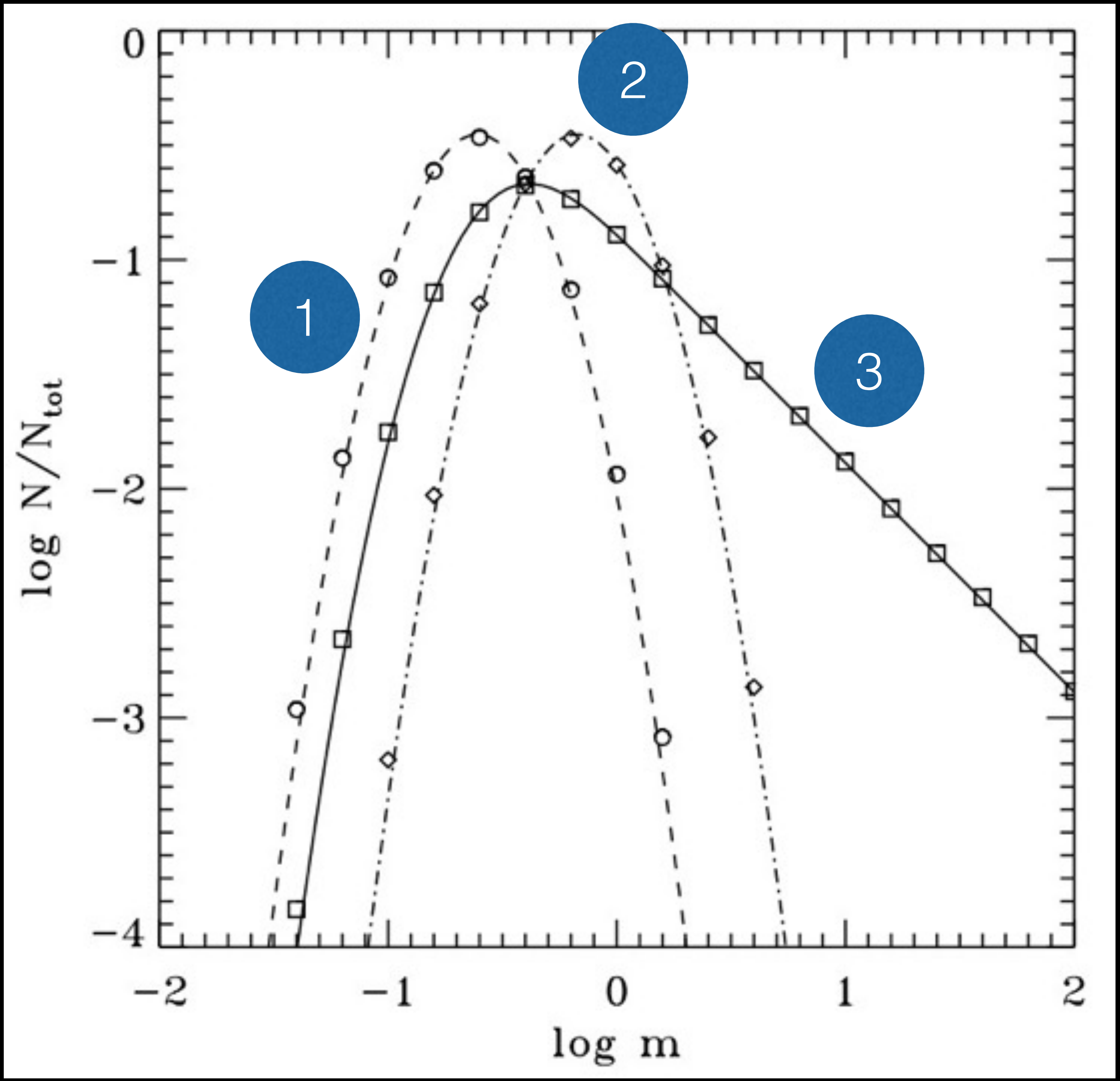
$$f(t) = \delta e^{-\delta t}$$

$$\int_0^\infty \frac{1}{\sqrt{2\pi}\,\sigma_0\, m} \exp\frac{-(\ln m - \mu_0 - \gamma t)^2}{2\sigma_0^2}\,\delta e^{-\delta t}\,dt = \frac{\alpha_0}{2}\exp\left(\alpha_0\mu_0 + \alpha_0^2\sigma_0^2/2\right) m^{-(1+\alpha_0)} \times \left(\operatorname{erfc}\left(\frac{1}{\sqrt{2}}\left(\alpha_0\sigma_0 - \frac{\ln m - \mu_0}{\sigma_0}\right)\right)\right)$$

MLP 3 parameters: μ_0 ,σ_0,α_0, : $\alpha_0 = \delta/\gamma$

Source: http://abyss.uoregon.edu/~js/ast122/lectures/lec13.html

Basu et al. 2015



1 Initial Lognormal distribution with mean μ_0 and standard deviation σ_0.

2 Exponential growth of masses results into mean $\mu_0 + \gamma t$

3 Modified Lognormal distribution for exponential distribution of accretion times

Basu and Jones 2004

Note: For $\alpha_0 >> 2$, MLP becomes lognormal.
and $\sigma_0 \sim 0$, MLP becomes power-law.

How is the MLP different?

MLP	Chabrier	Kroupa
Single Function.	Piecewise function with joining conditions.	Piecewise function with joining conditions.
Only 3 parameters.	4 parameters: 1 joining conditions.	5 parameters: 2 joining conditions.
Hybrid function modeling the entire mass regime.	Lognormal function probes sub-stellar and low-mass and power-law probes intermediate high mass.	Broken power-laws model over different mass regimes.
No joining conditions.	No physical explanation for the joining condition.	No physical explanation for the joining conditions.
Has some physical explanation.	No physical explanation for the power-law behaviour.	No physical explanation for the broken power-laws.

Basu et al. 2015

Where to test the model?

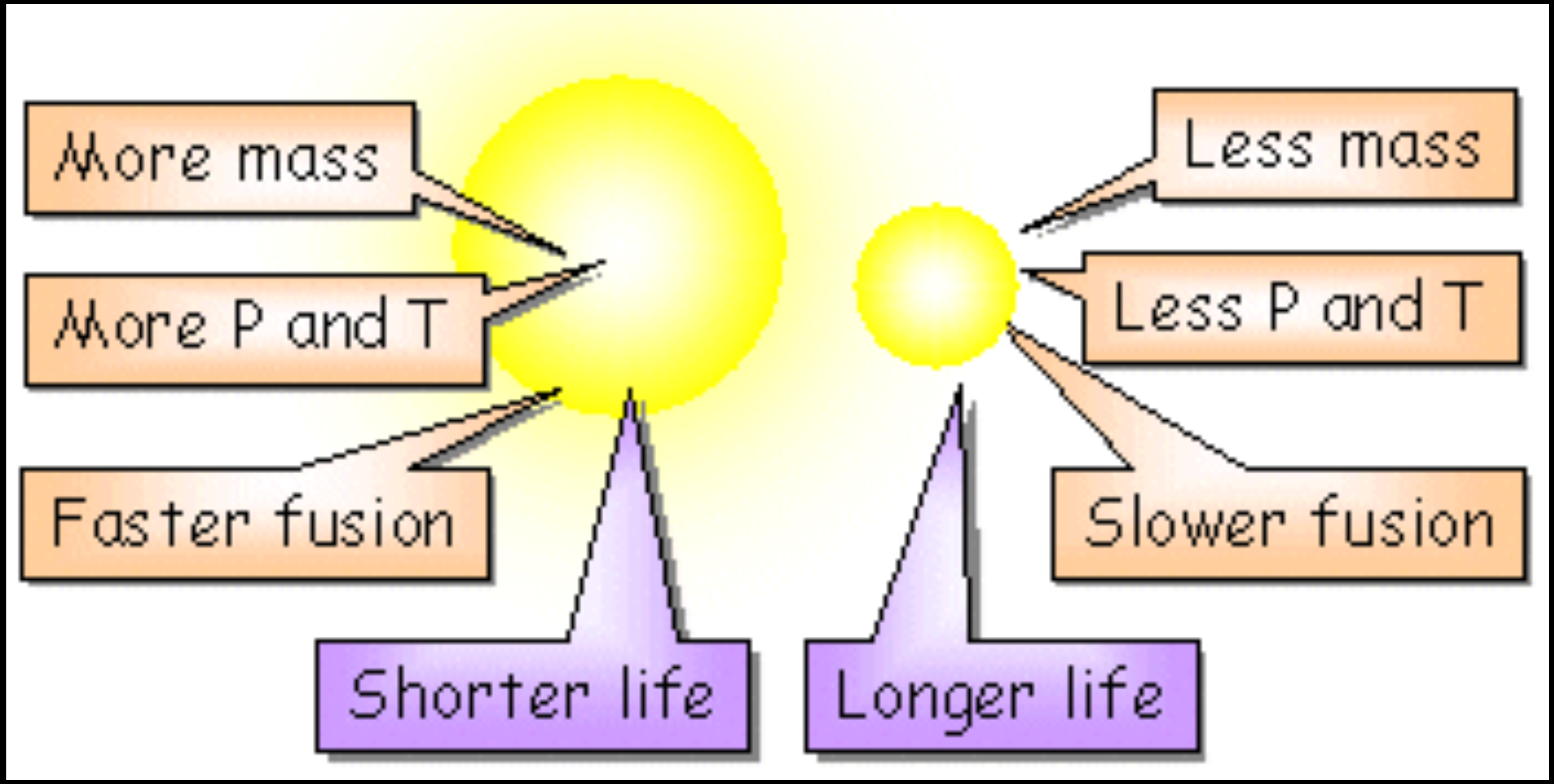
YOUNG AND POPULOUS STELLAR CLUSTERS

- **What is a stellar cluster?** an aggregate of stars that form the fundamental (same age, metallicity , and distance) building blocks of galaxies

- **Why Populous?** To obtain a statistically significant sample

YOUNG AND POPULOUS STELLAR CLUSTERS

- **Why Young?** A young SSP helps us to do model the high mass end of the mass function since a relatively young cluster still contains some high mass stars that are not yet evolved off the main sequence.

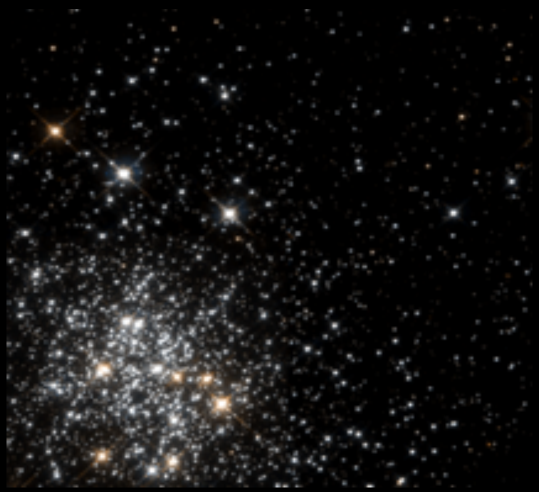


Modeling the Mass Function of NGC1711 as a case study using the MLP

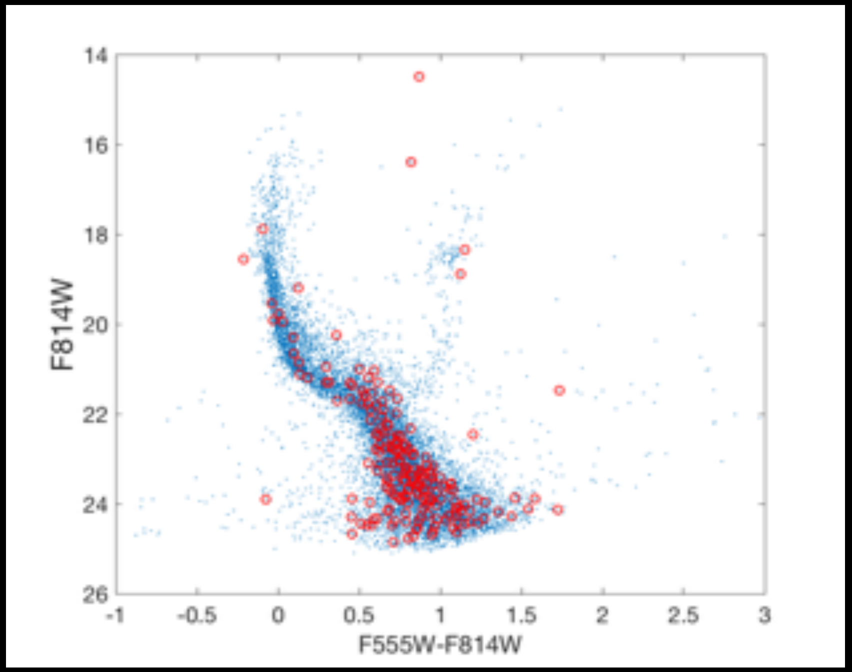
Objectives:

- **Deriving the MASS FUNCTION of NGC1711**
- **MLP Model Fitting**

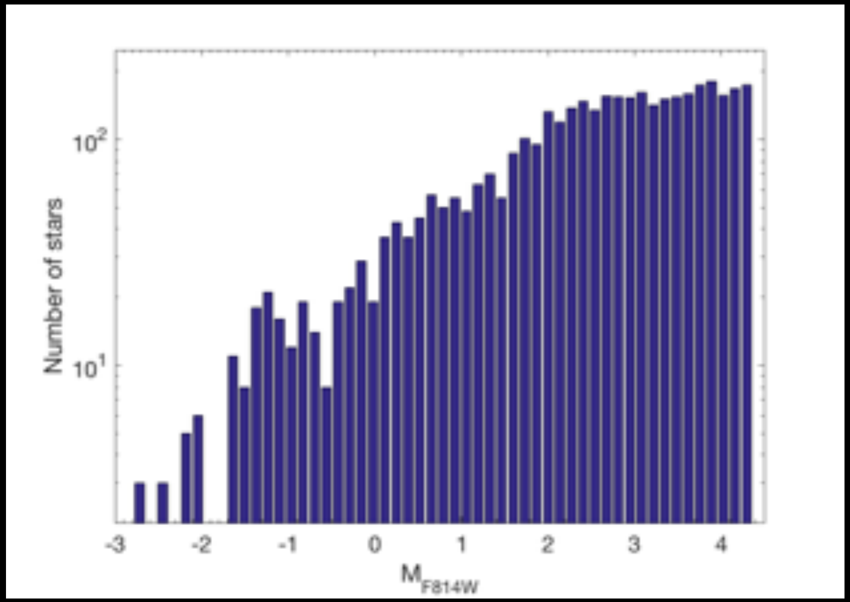
Deriving the MF of NGC1711



Resolved stellar population



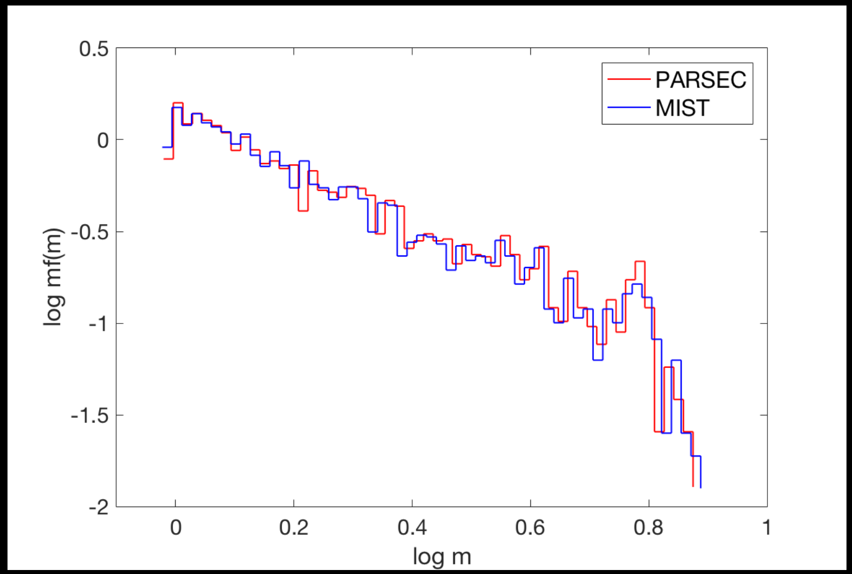
Completeness Correction and field star contamination



Luminosity Function (LF)

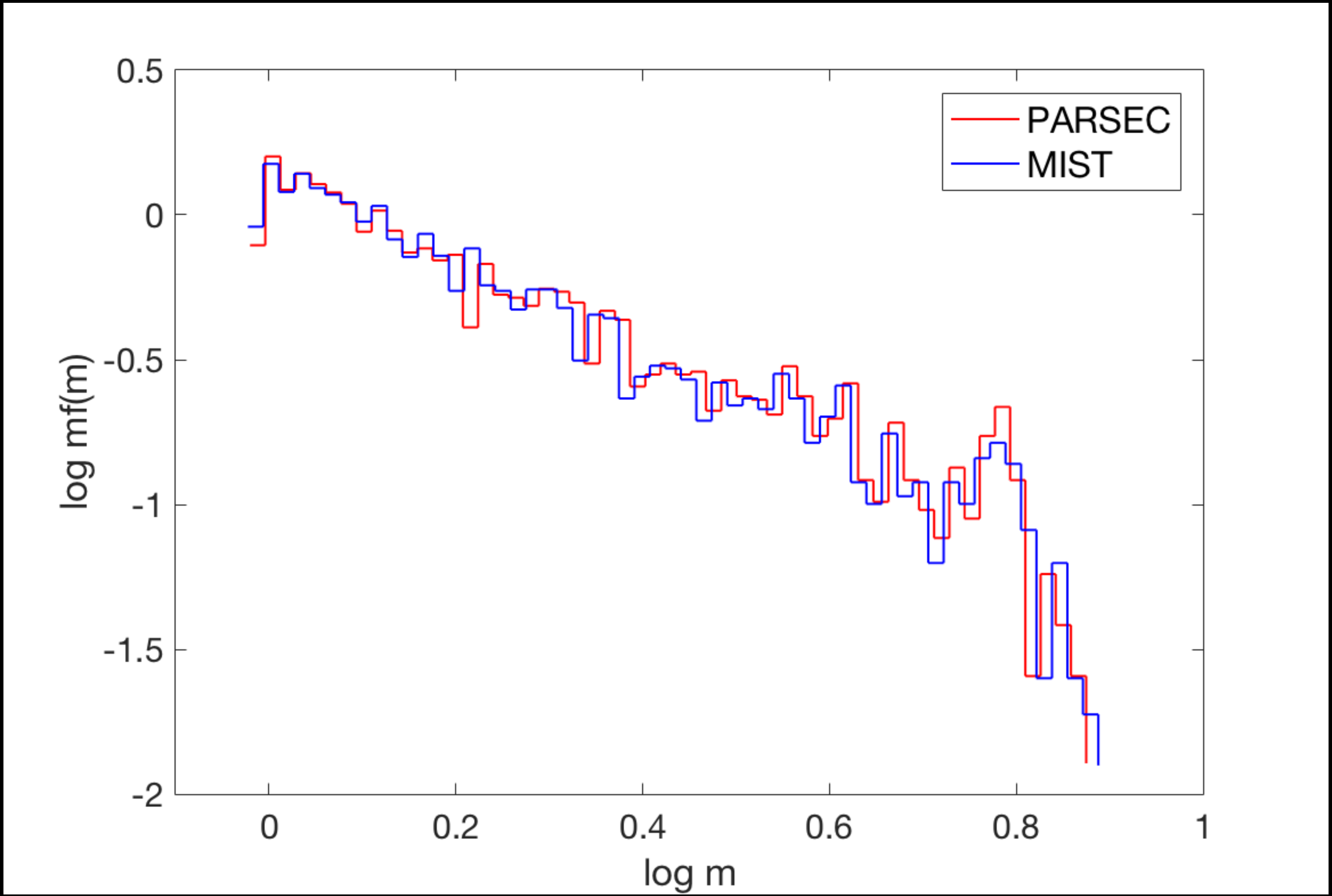
MLR

Mass-Luminosity Relation



Mass Function (MF)

Mass Function (MF)



MLP model fitting

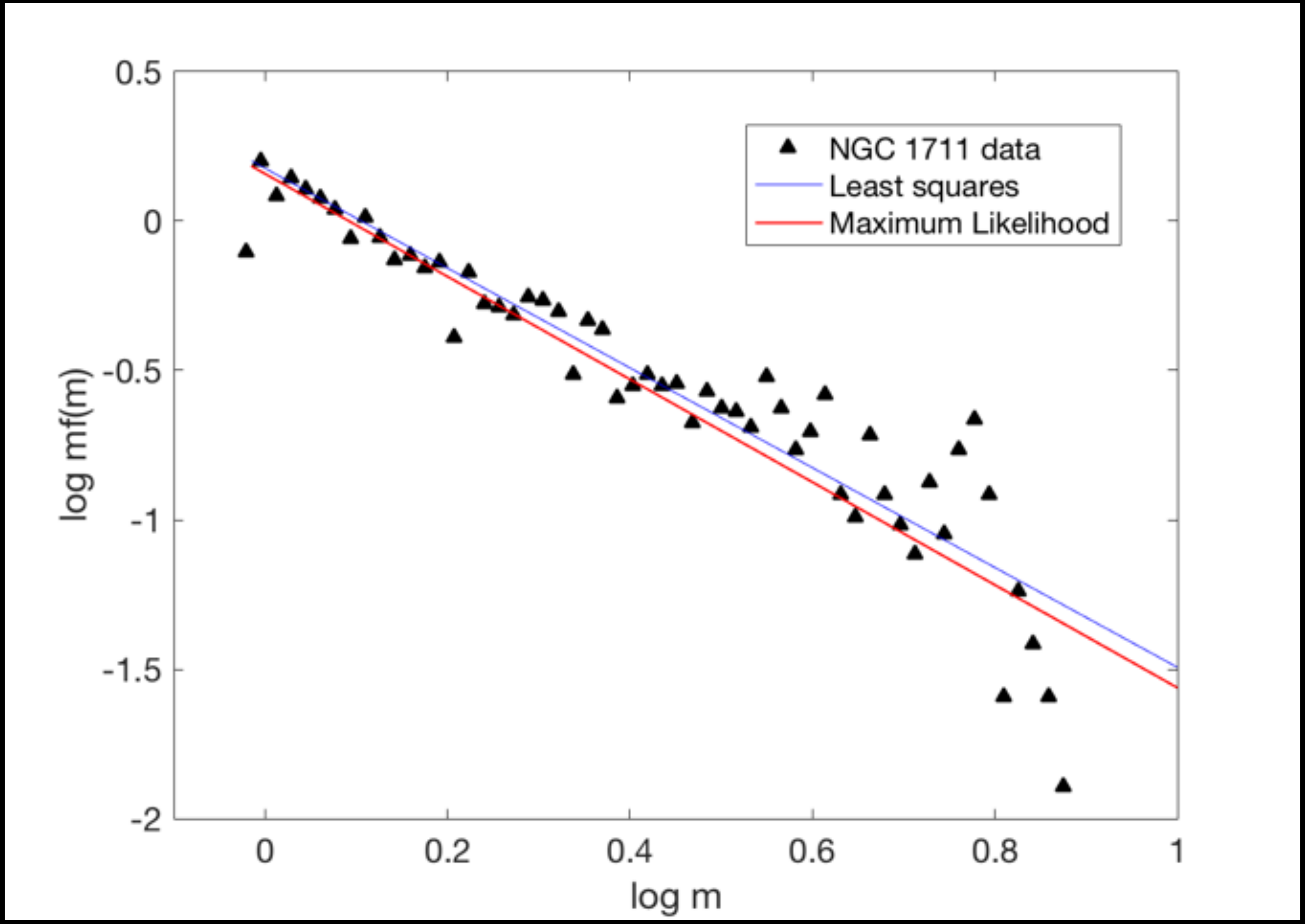


Figure 8. Fits to the MF found using PARSEC isochrones. The best fit MLP function using least squares is the blue line and has parameters $\alpha = 1.68$, $\mu_0 = -0.06$ and $\sigma_0 = 0.07$. The red line represents the best fit MLP function using maximum likelihood estimation and we find $\alpha = 1.72$.

Effect of a lognormal body

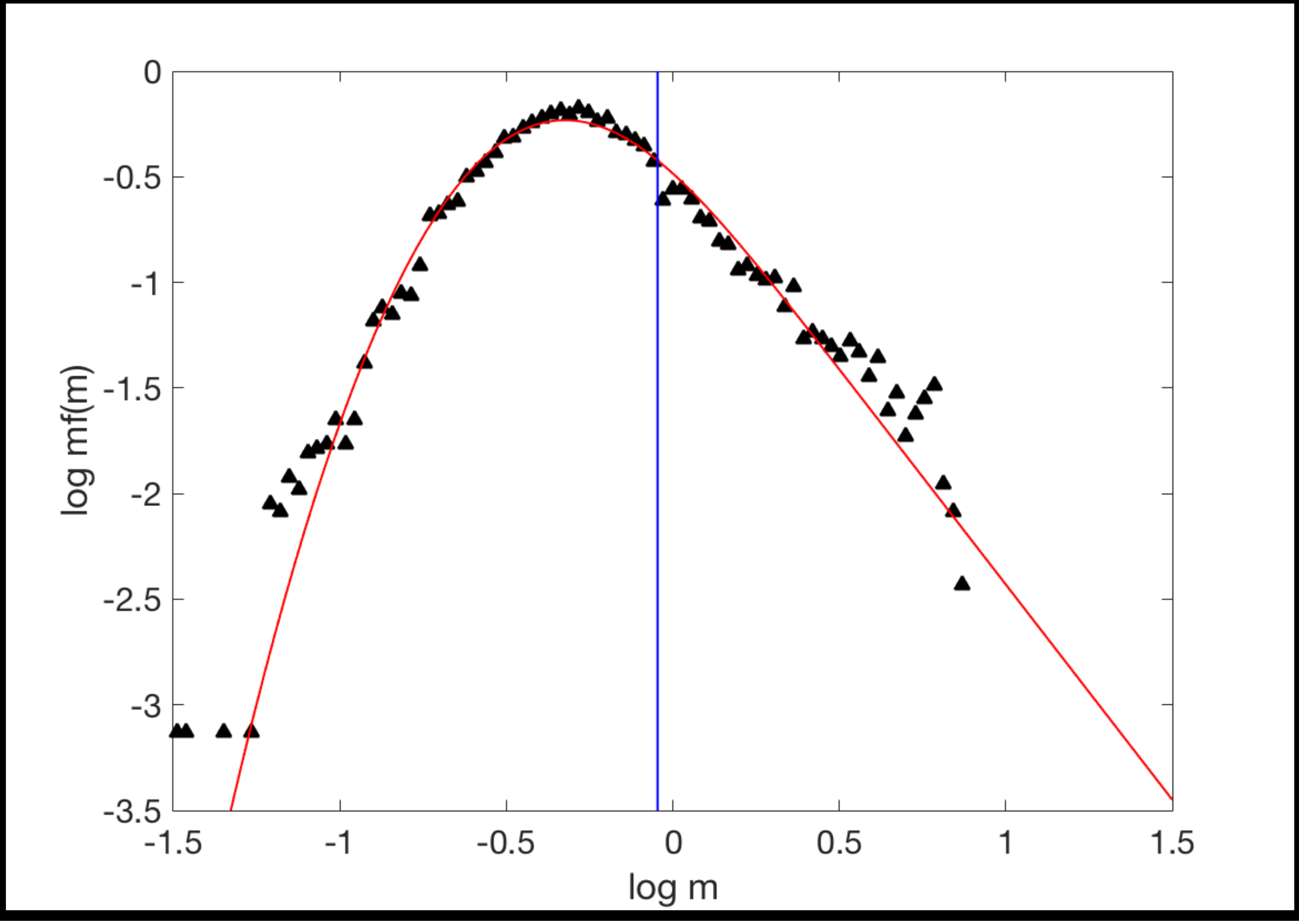


Figure 12. MLP fit to the entire stellar mass domain including the artificially generated data points from the Chabrier function. The best fit parameter values are $\alpha = 2.04 \pm 0.07$, $\mu_0 = -1.10 \pm 0.01$ and $\sigma_0 = 0.55 \pm 0.01$. The blue line represents the demarcation of the artificially generated data points from the NGC 1711 data points.

Summary

- We obtained the MF of NGC1711 from the LF using theoretical MLRs.
- We then used the MLP function to describe the MF of the stellar population.
- We find that the MF of NGC 1711 follows a pure power-law behaviour for masses above the completeness limit.
- We added a lognormal body to the NGC1711 data by random sampling from the Chabrier pdf.
- We then conclude that the absence or presence of a lognormal body can alter the measured slope of the power-law tail, making it either shallower or steeper.

Future Work

- Model Selection to compare different IMF models.
- Truncated MLP with finite accretion lifetimes.

THANK YOU ALL FOR COMING!

Big thank you to my stellar research group!